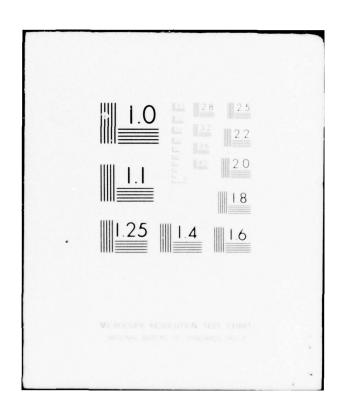
NATIONAL AVIATION FACILITIES EXPERIMENTAL CENTER ATL-ETC F/G 17/7 SIMULATION MODEL FOR AIR TRAFFIC CONTROL COMMUNICATIONS.(U)

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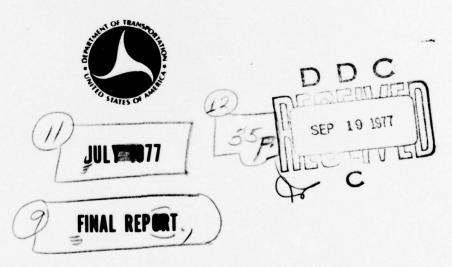


Report No. FAA/RD/77/69

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### SIMULATION MODEL FOR AIR TRAFFIC CONTROL COMMUNICATIONS

Robert/Mulholland



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#### **Technical Report Documentation Page**

| 1. Report No.<br>FAA-RD-77-69  | 2. Government Accession No.           | 3. Recipient's Catalog No.  |
|--|---------------------------------------|---|
| 4. Title and Subtitle SIMULATION MODEL FOR AIR TR  | AFFIC CONTROL COMMUNICATIONS          |   |
|  |                                       | Performing Organization Code     Performing Organization Report No. |
| 7. Author(s) Robert Mu   | 1ho11and                              | FAA-NA-76-30  |
| 9. Performing Organization Name and Address  | 15                                    | 10. Work Unit No. (TRAIS)   |
| Federal Aviation Administra<br>National Aviation Facilitie<br>Atlantic City, New Jersey O            | 11. Contract or Grant No. 061-221-100 |   |
|  |                                       | 13. Type of Report and Period Covered                               |
| 12. Sponsoring Agency Name and Address<br>U.S. Department of Transpor<br>Federal Aviation Administra |                                       | Final   |
| Systems Research and Develowashington, D.C. 20590  | 14. Sponsoring Agency Code            |   |
| 15. Supplementary Notes  |                                       |   |

LA Abstract

A computer simulation model designed to mimic second-by-second behavior of air/ground communications in an air traffic control sector is described. The model can be used to simulate any one of nine sector functions (e.g., high-altitude enroute, low-altitude transitional, radar-arrival control, etc.). The model exists as a computer program written in the GPSS V and FORTRAN IV languages. Input variables include aircraft arrival rate into sector, distribution of transmission length, distribution of number of transmissions in an air/ground exchange, etc. Response variables include sector aircraft loading, channel utilization, and communications delay. Model output can be obtained in the form of time series (e.g., minute-by-minute averages of channel utilization) exhibiting the dynamics of sector communications or simple averages of such series taken over several hours of simulated time.

SEP 19 1977

17. Key Words
Air Traffic Control Simulation
Computor Simulation
Simulation Modelling

18. Distribution Statement
Document is available to the public
through the National Technical Information
Service, Springfield, Virginia 22151

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price |
|--|--------------------------------------|------------------|-----------|
| Unclassified                           | Unclassified                         | 36               |           |

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#### INTRODUCTION

This report is a brief description of the main aspects of a computer simulation model designed to mimic air/ground communications in an air traffic control (ATC) sector. The model was developed through a joint effort of the National Aviation Facilities Experimental Center (NAFEC) and Princeton University to apply fast time simulation techniques and methods of time series analysis to ATC problems (references 1 to 4). The model has been validated with field data obtained in 1969 from the New York Control Area and additional information collected from the Houston Control Area in 1971. The model is presently available in a software package adapted for use in computer facilities at NAFEC. It can be used to simulate the following nine sector functions:

- HI = High-altitude enroute (radar controllers)
- LE = Low-altitude enroute (radar controllers)
- LT = Low-altitude transitional (radar controllers)
- GN = Ground control (tower controllers)
- LC = Local control (tower controllers)
- LG = Local ground control (tower controllers)
- DP = Radar departure control (IFR (instrument flight rules) Room radar controllers)
- AD = Radar arrival/departure control (IFR Room radar controllers)
- AR = Radar arrival control (IFR Room radar controllers)

In the following paragraphs, we describe the structure of the model, input variables, output variables, and some possible applications. More detailed descriptions can be found in references 2 and 3.

#### DISCUSSION

#### STRUCTURE OF COMMUNICATIONS.

Before proceeding to a discussion of the model, some formulation of the structure of air/ground communications is in order. An air/ground conversation normally consists of several transmissions (TR's) alternately initiated by pilot and controller. In keeping with earlier work, a whole conversation is referred to as a communication transaction (CT), so that a CT consists of one or more TR's. The reader is advised to keep the distinction between TR's and CT's in mind.

Obviously, while an aircraft is in sector, there are several CT's between controller and pilot as shown in figure 1. From the standpoint of communications, we adopt the attitude that an aircraft arrives at a sector with the beginning of the first CT with the ground and leaves the sector at the end of the final CT. Between these points in time, the communication between a single aircraft and ground usually consists of several CT's and gaps between CT's. We refer to these gaps as intercommunication gaps. Referring to figure 1, it is apparent

that the number of intercommunication gaps is one less than the number of CT's, so that specification of either automatically determines the other.

Figure 1 is essentially a picture of air/ground communications from the point of view of a pilot. Since there are usually many aircraft in sector simultaneously, the communications picture from the point of view of the controller is just a superposition of several diagrams like that of figure 1. This is demonstrated in figure 2 for the case where the maximum number of aircraft in sector between time 1 and 2 is two. Obviously, the gaps between CT's from the viewpoint of the controller are normally shorter than those experienced by the pilot of a single aircraft. In order to distinguish the former from the latter, we refer to the gaps experienced by the controller as intertransaction gaps. Thus, the pilot experiences intercommunication gaps, and the controller experiences intertransaction gaps.

#### STRUCTURE OF THE PROGRAM.

The simulation model exists as a computer program written in the GPSS V and FORTRAN IV languages. It is designed to mimic second-by-second behavior of sector air/ground communications over periods of time in the order of hours. A flow chart of the model is displayed in figure 3. As indicated by the blocks of the chart, the model performs nine basic operations. These are described below.

Aircraft arrivals (block 1) at the sector under investigation correspond to a sample function from a Poisson process. Stated another way, interarrival times are modeled as independent exponential variates with a common average (AMEAN) expressed in seconds. AMEAN is an input variable that specifies the average rate, in this case, 1/AMEAN aircraft per second, with which aircraft enter the sector.

When an aircraft arrives at the sector, it is assigned a number of CT's (block 2) by means of a random sample drawn from a negative binomial distribution with shifted origin and parameters P and K, i.e.,

K+r-2 
$$P^{K}(1-P)^{r-1}$$
; r=1,2,3,....  
K-1

For example, P and K might be estimated by the method of moments from historical data collected from a sector or group of sectors of the type being studied. In addition to the number of CT's, an incoming aircraft is assigned a mean intercommunication gap length (MGAP) (block 3). The natural logarithm of MGAP is generated from a normal distribution. The mean (XM) of this distribution is given by,

$$XM = A1 + A2XN$$

where N is the number of gaps, one less than the number of CT's, and Al and A2 are regression coefficients determined from historical records of mean gap length and number of gaps obtained for many aircraft passing through sectors

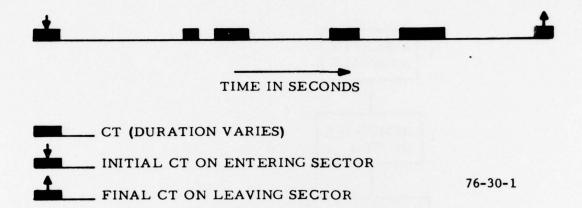


FIGURE 1. COMMUNICATIONS FROM PILOT POINT OF VIEW

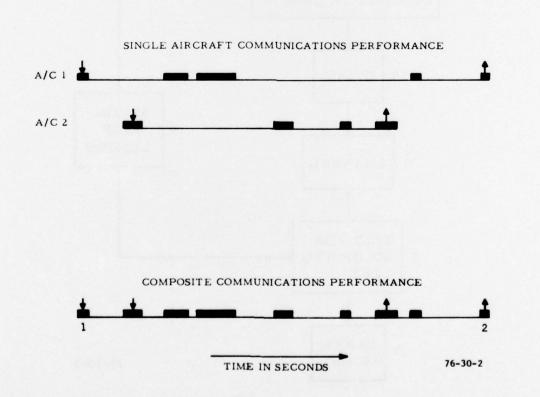


FIGURE 2. COMMUNICATIONS FROM CONTROLLER POINT OF VIEW

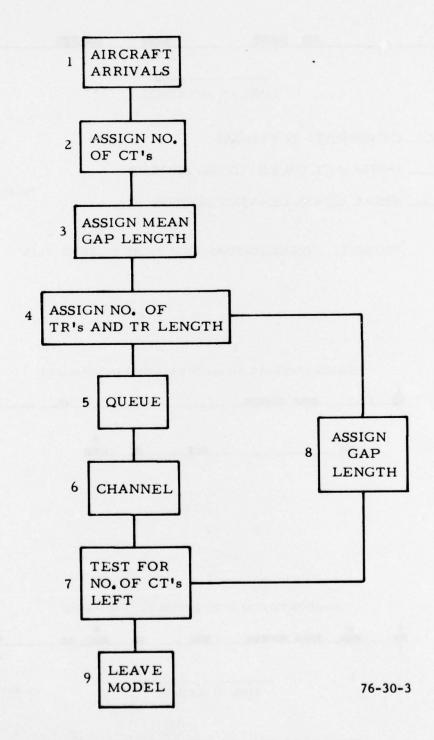


FIGURE 3. BLOCK DIAGRAM OF SIMULATION MODEL

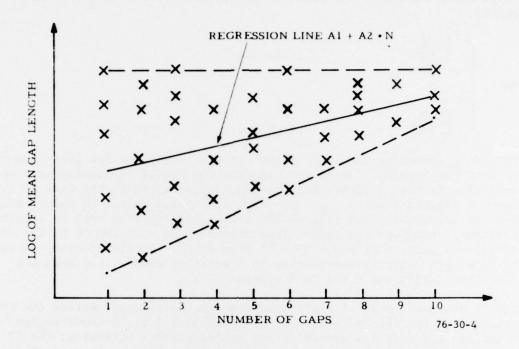
of the type being studied. The standard deviation (SD) is determined from the relationship,

SD = 
$$\begin{cases} \frac{\text{CU-XM}}{2.5758} & \text{if A2 } > 0 \\ \frac{\text{XM-CL}}{2.5758} & \text{if A2 } < 0 \end{cases}$$

where CU and CL are upper and lower bounds, respectively, of the logarithm of recorded mean gap lengths. These formulas are the result of examinations of historical records that indicate the existence of patterns in the data of the type shown in figures 4 and 5. The constant 2.5758 was chosen to insure that 99 percent of the probability mass of the normal distribution used to generate the logarithm of the mean gap length lies between 2 XM - CU and CU in the case where A2 >0, and between CL and 2 XM - CL when A2 <0. In either case, whenever sampling from this distribution results in a negative number, the mean gap length is arbitararily set equal to 1 second.

The first CT assigned to an aircraft commences as the aircraft enters the sector provided that the channel (block 6) is available, i.e., a conversation between another aircraft and ground is not taking place. Otherwise, the CT enters a queue (block 5) on a first-come, first-serve basis. As each CT is completed, the model ascertains whether or not all assigned CT's have taken place (block 7). If not, then an intercommunication gap length is randomly selected from an exponential distribution with mean MGAP seconds (block 8). If it is determined that the CT just completed is indeed the last of the assigned CT's, then the aircraft leaves the sector (block 9). At any rate, the end time of the most recently completed CT and the length of the subsequent intercommunication gap determines the start time of the next CT. Of course, in the event that the channel is busy when the CT is scheduled to take place, then the CT enters a queue and waits in turn for transmission service. When the start time of a CT is established, the corresponding CT length is determined in a two-step process (block 4). The first step of the process is to establish the number of TR's involved in the CT. This is accomplished by a random sample drawn from an empirical distribution based upon data collected from one or more sectors of the type being investigated. Thereafter, the length of each TR is obtained as a random sample drawn from a gamma distribution with parameters  $\alpha$  and  $\lambda$ , i.e.,

$$\frac{1}{\lambda \alpha \Gamma (\alpha)}$$
 t  $^{\alpha-1}$   $\epsilon$   $^{-t/\lambda}$  0 \infty



XM VERSUS N IN CASE WHERE A2 > 0

FIGURE 5.

The parameter values are obtained from two master equations (i) and (ii). These are,

(i) 
$$\alpha \cdot \lambda$$
 = 3.70 - 56.88/AMEAN (General)  
= 2.97 - 15.12/AMEAN (LC)

(ii) 
$$(\alpha - 1)\lambda = 2.0$$
 (General)  
 $(\alpha - 1)\lambda = 1.7$  (GN)  
 $(\alpha - 1.13)\lambda = 1.72$  (DP)  
 $(\alpha - 1.5)\lambda = 1.0$  (AD)

where, as already indicated, AMEAN is the mean aircraft interarrival time in seconds. Thus, if the LC sector function is to be simulated, then appropriate values for  $\alpha$  and  $\lambda$  are obtained from the two equations,

$$\alpha \cdot \lambda$$
 =2.97 - 15.12/AMEAN  
( $\alpha$  -1)  $\lambda$  =2.0

A listing of the program available at NAFEC is provided in the appendix. This listing together with the description of the program structure presented here is merely intended to provide the reader with some general idea of the existing simulation capability. Further details including possible modifications, variations in the manner in which inputs can be supplied to the model, justification of model formulation, etc., can be obtained from the cited references.

#### INPUT VARIABLES.

As described, the model is characterized by 10 input variables, namely, AMEAN, P, K, A1, A2, CU, CL,  $\alpha$ ,  $\lambda$ , and an empirical distribution for the number of TR's contained in one CT. AMEAN prescribes the mean interarrival time between successive aircraft entering the sector. P and K determine the distribution of the number of CT's, or equivalently, the number of intercommunication gaps, experienced by an aircraft as it passes through the sector. The four parameters, A1, A2, CU, and CL, determine the distribution of gap length. Finally,  $\alpha$ ,  $\lambda$ , and the empirical distribution of the number of TR's in a CT specify the distribution of CT length.

In an application of the model, all input parameters except AMEAN might be assigned fixed values to represent a particular sector in some center such as Houston or New York. Then several simulations could be performed corresponding to decreasing values of AMEAN to ascertain the effect of increasing aircraft arrival rate on sector communications. By the same token, AMEAN could be held constant and the effect of some other input parameter on sector communications observed.

#### RESPONSE VARIABLES.

Three basic response variables are observed during each second of simulated time. They are the sector aircraft loading (i.e., the number of aircraft that have been handed off to the sector, but have not yet been handed off by the sector), the state of the air/ground channel (i.e., busy or idle), and the queuing state of each aircraft in the sector (i.e., in queue or otherwise). An aircraft is defined to be in queue if either the controller or pilot desires to converse with the other, but is prevented from doing so by virtue of the fact that a conversation is presently taking place between another aircraft and ground. Thus, insofar as the computer simulation is concerned, the third response variable measures the lag between the instant that the first TR of a CT is scheduled to occupy the channel and the time that it is actually carried by the channel. In this sense, the queuing state of the channel represents delay in the transfer of information between air and ground.

The next few paragraphs discuss each basic response variable in greater detail in the context of a specific simulation run. Values selected for input parameters in this example are as follows:

AMEAN=86 seconds P=0.495 K=3.88 A1=4.336 A2=0.032 CU=6.0 CL=3.1

The example experiment was designed to simulate New York LC sector 510. As a result,  $\alpha$  and  $\lambda$  were determined from the second of master equations (i) and the first of master equations (ii). Moreover, the empirical distribution of the number of TR's in a CT was determined from historical records obtained from sector 510. The simulation was allowed to run for 1 hour of simulated time prior to the accummulation of any data in order to dissipate the influence of transient phenomena generated by boundary conditions that exist at the beginning of the experiment. Thereafter, data were gathered for 2 hours of simulated time. Consequently, results obtained for the 2-hour observation period can be viewed as representative of sample functions drawn from stationary random phenomena. Of course, in the event that a transient effect persists or that values assigned to input parameters result in an explosive situation, then the underlying stochastic processes are far from stationary. However, such circumstances are usually reflected by very definite trends in the time series generated by one or more of the basic response variables during the 2-hour observation period.

#### CHANNEL UTILIZATION AND AIRCRAFT LOADING.

From observation of the first response variable, it is possible to compute the number of aircraft in sector per second averaged over each minute of simulated time. The average over the t th minute of simulated time is represented by  $n_t$ , and there are 120 such averages; namely,  $n_1$  through  $n_{120}$ . These are illustrated for LC sector 510 in figure 6. The number of aircraft in sector averaged over 2 hours of simulated time is just the arithmetic mean of  $n_1$  through  $n_{120}$ . For example, the 2-hour average corresponding to figure 6

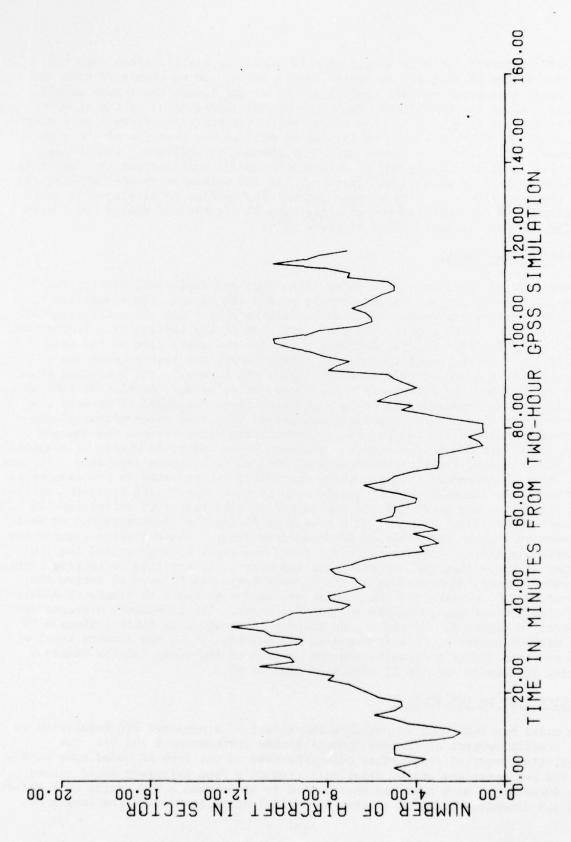


FIGURE 6. SIXTY-SECOND AVERAGE AIRCRAFT LOADINGS

is 5.983 aircraft per second, and this is listed in table 1 along with the maximum number of aircraft in sector during any second of simulated time and the total number of aircraft handled by the sector during the 2-hour sample period. Along the same lines, second-by-second observations of the channel state, busy or idle, can be used to determine the fraction of the 2-hour sample period during which the channel is busy as well as the fraction of the t th minute of simulated time during which the channel is utilized. The latter fraction is denoted by  $c_t$  and is called the channel utilization; the former is called the average channel utilization. The 120 values of channel utilization are shown in figure 7. The average channel utilization is displayed in table 1 together with the total number of air/ground CT's generated during the 2-hour period, and the average length of these CT's.

#### COMMUNICATIONS DELAY.

We now turn to the last of the three basic response variables; namely, the aircraft queuing state. As an aircraft enters the sector, the simulation software assigns the number of CT's that are to take place while the aircraft is subject to sector control and the start time of the initial CT. Thereafter, upon completion of a CT, the software schedules the start time of the next CT, if any. In the event that two or more CT's vie for simultaneous use of the channel, a queue of CT's is established. As a result, the scheduled start time of a CT and the actual start time need not coincide. As already indicated, an aircraft is considered to be in queue whenever a scheduled CT between that aircraft and ground is in queue. Thus, second-by-second observations of the aircraft queuing state can be used to compute the delay between the instant that the CT would take place in the presence of an unlimited number of channels and the time that the air/ground channel does in fact become available. Obviously, the delay is dependent upon the queue discipline incorporated in the simulation software. In the case of the example used in this report, the first-in, firstout discipline was used, and the corresponding time that a CT waited for the channel was averaged over all CT's generated during the simulation run as well as over only those CT's that experienced some delay. These average figures are recorded in table 1 together with the total number of CT's generated and the number of those that did not encounter any delay. In addition to waiting times, second-by-second observations of the queuing state can be used to derive the number of CT's waiting for the channel averaged over the t th minute of simulated time and the entire sample period of 2 hours. The 120-minute averagem are plotted in figure 8, and the 2-hour average is provided in table 1 along with the maximum number of CT's in queue in any second. Thus, the average level of the graph in figure 8 is .426, and the maximum of the queue lengths observed during successive seconds of simulated time is 6.

#### APPLICATIONS OF THE MODEL.

The model has been used to evaluate the effect of a proposed FCC regulation on air traffic control air/ground communications (references 4 and 5). The regulation required transmitter identification in the form of coded tone bursts at the beginning and end of pilot initiated TR's from privately owned aircraft. The duration of each tone was anticipated to be between a few tenths of a second and 1.5 seconds. This, of course, is equivalent to increasing the length of

| S - SEC STON PER STON |  | CASE GOOF CIPILATION MUGEL FOR ATC VECAAL COMMUNICATIONS SYSTEM ************************************ |  | (S) INTERCHAUMICATION GAP LENGTHS ARE A FUNCTION OF TRANSACTIONS DEG ALCOPART SINTH ATTER FESSIONSE - 2 HOUR ANALYSTS (1) RECORE ALGORARY TOALTHA | 2 | AVENATE CHATMEL UTTLIZATION = .469 TOTAL MUMARM OF TRANSACTIONS = 476 AVENAGE FEBRIT OF TRANSACTIONS = 7.932 SECCNDS | Sq. | 267CFNT UF ZFRU FYTTIES (N'TH-WAT TAG) = 191<br>AVEAGE RUNNER OF AINCEAFT 'N OTEUF = .426 |
|--|--|--|--|---|---|--|-----|---|
|--|--|--|--|---|---|--|-----|---|

TABLE 1. GPSS SIMULATION MODEL FOR ATC VERBAL COMMUNICATIONS SYSTEM

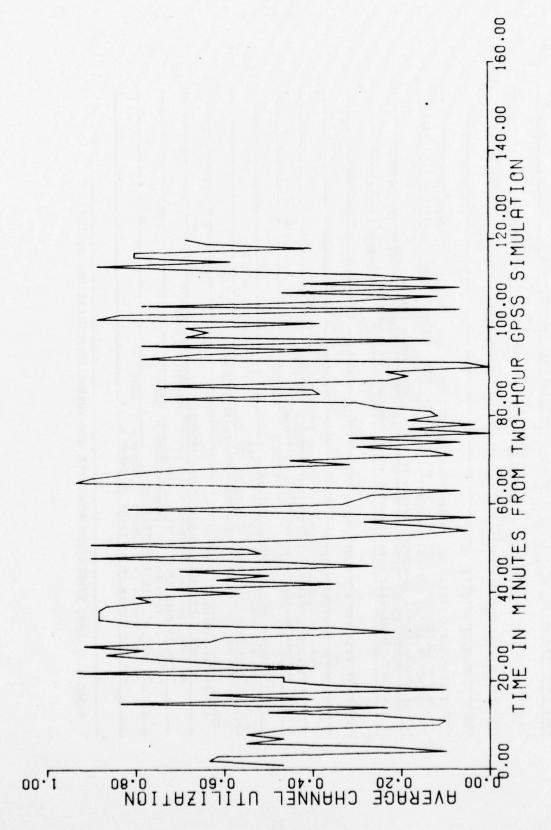
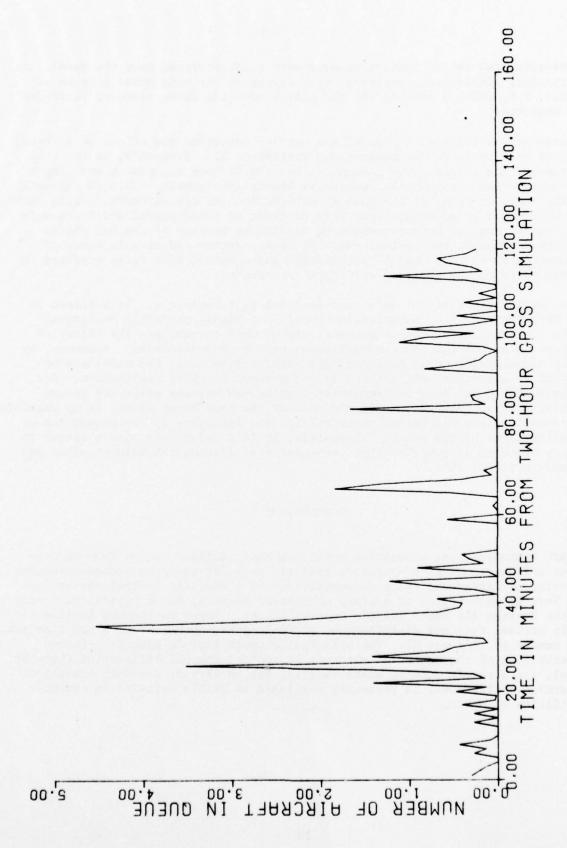


FIGURE 7. SIXTY-SECOND AVERAGE CHANNEL UTILIZATION



pilot-initiated TR's. These increases were incorpororated into the model. In particular, simulation experiments were conducted for tone burst lengths of 0, 0.5, 1.0, and 1.5 seconds and the effect upon the basic response variables was observed.

In another application, the model was used to determine the effect of aircraft arrival rate on sector communications (reference 4). Obviously, as the rate increases, the intensity of communications traffic and the number and length of communication delays will eventually become intolerable. This is reflected by the model in terms of the channel utilization and the aircraft queuing state. The problem is to ascertain that rate or range of rates beyond which adequate air traffic control service cannot be maintained because of demands placed upon the communication system. In this sense, sector capacity in terms of aircraft arrival rates can be established and compared with rates expected in future years on the basis of air traffic forecasts.

Other applications of the model can be found in reference 4. It suffices to say here that the ATC communication model is a highly versatile analytical tool. In its present form, the model can be used to evaluate the effect of changes in any of the input variables on sector communications. Moreover, by using historical records obtained from operating sectors, the model may be adapted to situations not covered by its present software realization. For example, the model does not presently distinguish between pilot and ground initiated TR's. However, as in the case of the tone burst study, it is possible to determine from historical records (e.g. see reference 1) the proportion of TR's initiated by the pilot. Thereafter, it is a relatively simple matter to modify existing software so that the model does distinguish between pilot and ground-initiated TR's.

#### **CONCLUSIONS**

An ATC communications simulation model has been realized in the form of computer software. It is a versatile analytic tool for studying second-by-second behavior of air/ground voice communications in a specific control sector, say New York LC sector 510, or a class of control sectors, say a typical HI sector in the Houston Air Route Traffic Control Center. Input parameters include air-craft arrival rate and distributions of TR lengths, number of CT's per aircraft, and number of TR's per CT. The principal outputs include second-by-second observations of the number of aircraft in sector, channel utilization (busy or idle), and aircraft queuing state (waiting for an already occupied channel or otherwise.) The model is presently available as usable software in computer facilities at NAFEC.

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- 4. Hunter, J. S. and D. -A. Hsu, <u>Applications of the Simulation Model for Air Traffic Control Communications</u> (Accepted for publication as an official report prepared for the U.S. Department of Transportation, Federal Aviation Administration, System Research and Development Service, Washington, D.C.), April 1976.
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APPENDIX

PROGRAM LISTING

```
" REAL COCATE COM. 30000, HMS. 12
******
                              FUNC 110N S
                                          *****
TRET FUNCTION RN1,013
.195,1/.485,2/.741,3/.853,4/.925,5/.958,6/.978,7/.985,8
.991,9/.993,10/.996,11/.998,12/1.13
                              STORAGES
**********
SCTR STORAGE
                150
********
                              MATRICES
                                         *****
      MATRIX
                                     NAC IN SECTOR
1
                 MH,600,1
      MATRIX
                                     ARRIVAL TIMES
                 MH,120.1
                                     CT'S PER AIRCRAFT
      MATRIX
                 MH,120,1
                                     TR'S PER CT
      A.V L L I X
                 MH,400,1
      MATRIX
                 MH,1200,1
                                     TR LENGTHS
                                     CT LENGTHS
      MATRIX
                 MH,400,1
7
                 MH,400,1
      MATRIX
                                     INTERCOM. GAP LENGTHS
8
      MATRIX
                 MH,120,1
                                     AC TIME IN SECTOR
                                     DEPARTURE TIMES
      MATRIX
                 MH,120,1
10
      MATRIX
                 MH,600,1
                                     NAC IN QUEUE
      MATPIX
11
                 MH,400,1
                                     QUEUEING TIMES
      MATRIX
                                     CHANNEL STATUS
12
                 MH,600,1
*******
                              VARIABLES
                                          ******
LAMDA FVARIABLE
                1.0/(0.97-15.12/XL SAMEAN)
ALPHA EVARIABLE (1.0/XLSPGAM2+2.0) *XL $PGAM2
                              SAVEVALUES
                                          ****
                 XL $PNR1 ,3.88
      INITIAL
                                     K FOR CT/AC
       INITIAL
                 XL $ PNR 2 , 0 . 495
                                     P FOR CT/AC
                 XL . PGAP1 . 4.336
      INITIAL
                                     AT FOR GAP LENGTHS
      INITIAL
                 XL . PGA P2 , 0 . 032
                                     AZ FOR GAP LENGTHS
                                     COMPILING ONLY
      INITIAL
                 XL SPNST1,0.0
      INITIAL
                                     COMPILING ONLY
                 N.O.STRNASIX
                 XL .PGAM1 .O.O
      INITIAL
                                     COMPILING ONLY
      INITIAL
                 XL $PGAM2 , O . O
                                     COMPILING ONLY
      INITIAL
                 XL SAMEAN, 86.00
                                     MEAN INTERARRIVAL TIME
                                     SECTOR NUMBER
      INITIAL
                 XH10,510
      INITIAL
                 XF $ ONF , 48949
                                     RANDOM NUMBER GENERATORS
       INITIAL
                 XF$TW0,25701
      INITIAL
                 XF ! THRFF , P5261
       INITIAL
                 XF$FOUR,52693
      INITIAL
                 XF 1F 1VF , 2529
       INITIAL
                 XF . SIX , 18623
      INITIAL
                 XF 1 SEVEN , 44035
      INITIAL
                                     MEAN GAP LENGTH
                 XL SMGAP, O.O
      INITIAL
                 XF . YAC , O
                                     # OF CTS PER AC
      INITIAL
                 XFSTIMF,0
                                     CT LENGTHS
      INITIAL
                 XF ITRL,0
                                     TR LENGTHS
                                     INTERCOM. GAPS
      INITIAL
                 XFSTGAP,0
      INITIAL
                 XF SMTR , O
                                     INTERARRIVAL TIMES
      INITIAL
                 XFSIAT,0
                                     SECOND COUNTER FOR MATRIX
     INITIAL
                 XF 1C DUNT , O
      INITIAL
                                     MATRIX UNIT COUNTER
                 XH1,10
```

```
XH2,22
                                        STAT OUTPUT UNIT COUNTER
       INITIAL
       INITIAL
                  XH3-XH9,0
                                        MATRIX OUTPUT COUNTERS
       INITIAL
                  MH1(1-600,1),0 ·
                                        INITIAL MATRICES
                  MH2(1-120,1),0
       INITIAL
                  MH3(1-120,1),0
       INITIAL
       INITIAL
                  MH4(1-400,1),0
       INITIAL
                  MH5(1-1200,1),0
                  MH6(1-400,1),0
       INITIAL
       INITIAL
                  MH7(1-400,11,0
       INITIAL
                  MHP(1-120,1),0
       INITIAL
                  MH9(1-120,1),0
       INJTIAL
                  MH10(1-600,1),0
       INITIAL
                  MH]](1-400,1),0
                  MH12(1-600,1),0
       INITIAL
*******
                                SIMULATION
                                              SIMILATE
* DETERMINE GAMMA PARAMETERS AT REGINNING OF SIMULATION
                  * * * 1 , 25 , 0
       GENERATE
       SAVEVALUE
                  PCAMZ, VSLAMDA, XL
       SAVEVALUE
                  PGAMI, VSALPHA, XL
       TERMINATE
* GENERATE POISSON ARRIVALS AT SPECIFIED RATE
       GENERATE
                  ,,,,15,4PF,1PL
      GATE LR
       LOGIC S
       HFL PR
                  EXPON, AMFANSXL, IATSXF, SEVENSXF, ONFSXF, TWOSXF, FOURSXF
       ADVANCE
                  XF 5 ] A T
       FUCIC &
                                        MARK TIME OF ARRIVAL .
       MARK
* SAVE AND TARULATE TIME OF ARRIVAL
       ASSIGN
                  3,4C1, PF
       SAVEVALUE 3+ ,1 ,XH
                                        ARRIVAL COUNTER
       MSAVEVALUE 2,XH3,1,C1,MH
                                        TABULATE ARRIVAL TIME
       FNTER
                                        ENTER THE SECTOR
* DETERMINE THE NUMBER OF CT FROM A NEG. BIN. DSN. WITH SHIFTED ORIGIN
                  SURNR1, PNR1 $XL, PNR2 $XL, XAC $XF, ONE $XF, TWO $XF, THREE $XF
       HEL PR
       SAVEVALUE
                  XAC+,1,XF
       MSAVEVALUF 3, XH3, 1, XF$XAC, MH
                                        TABULATE CTS PER AIRCRAFT
                  1,XFSXAC,PF
       ASSIGN
* GENERATE MEAN GAP LENGTH FOR ENTERING AIRCRAFT, AS A FUNCTION
  OF THE NUMBER OF CIS ASSIGNED.
       SAVEVALUE
                  XAC-,1,XF
                                        MUMBER OF GAPS
                  GAPM, PGAP1 $XL , PGAP ; $XL , XAC $XF , MGAP $XL , TWO $XF , THREE $XF
       MEL PR
       ASSIGN
                  1 . XL SMGAP , PL
       TRANSFER
                                        SKIP GAP FOR FIRST CT
                  ,SCTS
```

```
* ADVANCE INTERCOMMUNICATION GAP TIME, THEN ENTER OUFLE.
   SEIZE CHANNEL WHEN AVAILABLE.
                  XF STGAP
                                         WAIT TIL GAP COMPLETED
SGAP
      ADVANCE
                   4PF
                                         MARK TIME IN OUFUE
SCTS
       MARK
                  SCTR
       OUFUE
       GATE LR
       LOGIC S
                  CMTRL
                                         SEIZE THE CHANNEL
SCTR
       SFIZE
       DEPART
                  SCTR
* TABULATE TIME IN OUFUE
       SAVEVALUE 9++1 .XH
                                         NUMBER OF QUELLEING TIMES
                                         TABULATE TIME IN OUFUE
       MSAVEVALUE 11, XH9, 1, MP4PF, MH
 GENERATE CT LENGTH
     TR'S PER CT FROM EMPIRICAL DISTRIBUTION
     TR LENGTHS FROM CAMMA DSN.
       SAVEVALUE TIME , O , XF
       SAVEVALUE NTR , FNSTRCT , XF
                                         TR'S PER CT
       SAVEVALUE 4+ ,1 , XH
                                         COUNT OF CT'S
                                         TARULATE TR'S PER CT
       MSAVEVALUE 4, XH4, 1, XF SNTR, MH
       ASSIGN.
                   2.XF SNTR , PF
                   MSSG, PGAM1 $XL, PGAM2 $XL, NTR $XF, TRL $XF, FIVF $XF, SIX $XF
       HFL PB
       SAVEVALUE 5+,1,XH
                                         COUNT OF TRIS
       MSAVEVALUE 5. XH5.1. XF STRL , MH
                                         TABULATE TR LENGTHS
       SAVEVALUE TIME+, XF STRL , XF
                                         ADD TR LENGTH TO CT LENGTH
       LOOP
                  2PF,STRL
                                         LOOP FOR MORE TR'S IN CT
       MSAVEVALUE 6, XH4, 1, XFST IMF, MH
                                         TABULATE CT LENGTHS
* HOLD CHANNEL FOR LENGTH OF CT. THEN FREE CHANNEL.
                                         WAIT TIL CT COMPLETED
                  XF STIME
       ADVANCE
* IMPOSE MANDATORY GAP OF 1 SECOND REFORE CHANNEL IS AVAILABLE.
                  1,TFST1
       SPLIT
       RFLFASE
                   CNTRL
                                         FREE CHANNEL
       APVANCE
                                         KEEP CHANNEL CLEAR FOR 1 SECOND
                   1
       LOGIC R
                   2
       TERMINATE
A GENERATE INTERCOMMUNICATIONS GAP FROM EXPON. DSN. GIVEN MEAN
  GAP LENGTH FOR THE AIRCRAFT.
# GAPS MUST BE AT LEAST ONE SECOND AND NO MORE THAN 700.
TESTI TEST NE
                  PF1,K1,SKIP8
                                         NO GAP IF NO CT'S REMAIN
       SAVEVALUE
                  MGAP, PL1, XL
                                         RETRIEVE MEAN GAP LENGTH
 TEST2 HFL PR
                   EXPON, MGAPSXL, TGAPSXF, THREESXF, TWOSXF, FOURSXF, FIVESXF
                   XF $TGAP , 700 , TFST2
                                         IF OVER 700, TRY AGAIN
       TEST LE
                   XF STGAP , 1 , SA VEA
       TEST LE
                                         IF LESS THAN 1, SET TO 1
       SAVEVALUE TOAP, 1, XF
 SAVES SAVEVALUE
                   6+,1,XH
                                         COUNT GAPS
       MISAVEVALUE 7, XH6, 1, XFSTGAP, MH
                                         TARLLATE GAP LENGTHS
```

```
IPF , SGAP
                                      LOOP TO MAKE MORE CT'S
* TARULATE TIME IN SECTOR IF AIRCRAFT ENTERED DURING SAMPLE PERIOD
SKIPR TEST G
                PF3,3600,SKIP1
      SAVEVALUE 7+,1,XH
                                      COUNT OF TIMES IN SECTOR
      MSAVEVALUE 8, XH7, 1, M1, MH
                                      TABULATE TIMES IN SECTOR
* TARJILATE DEPARTURE TIMES
SKIP1 SAVEVALUE 8+,1,XH
                                      COUNT OF DEPARTURES
      MSAVEVALUE 9, XH8, 1, C1, MH
                                      TARIJLATE DEPARTURE TIME
* LEAVE THE SECTOR
      I FAVE
      TERMINATE
                                          ****
   *****
                              MAC STATS
          THIS SECTION OF THE PROGRAM KEEPS TRACK OF THE NUMBER OF
     ATRCRAFT IN THE SECTOR IN FACH SECOND AND HAS THE DATA PUNCHED
     FOR LATER ANALYSIS.
      GENERATE
                 1,,3601,,2,0
                                      SECOND TIMER
      SAVEVALUE COUNT+, K1 , XF
                                      COUNT SECONDS
      MSAVEVALUE 1.XFSCOUNT,1.51,MH
                                      TABULATE NAC IN SECTOR
      MSAVEVALUE 10, XF COUNT, 1, 01, MH
                                      TARILLATE NAC IN QUEUE
       MSAVEVALUE 12, YESCOUNT . 1 . F1 . MH TARULATE CHANNEL STATUS
      TERMINATE
      GENERATE
                                      TEN-MINUTE TIMER
                 600,,4200,,1,0
                 1+,K1,XH
      SAVEVALUE
                                      MATRIX DUTPUT UNIT
                 TIME5, MH1(1,1), MH10(1,1), MH12(1,1), XH1, XF$ CNE, XF$ TWO
      HEI DE
                                      SET SECOND COUNT TO ZERO
      SAVEVALUE
                 COUNT, KO, XF
       TERMINATE
                                      HOUR TIMER
      GENERATE
                 3600,,7200,,3,0
       SAVEVALUE
                 2+, K1 , XH
                                      STAT OUTPUT UNIT
                 PASSZ, XH2, XF $ONE, XF $T WO, XF $THREE, XF $FOUR, XF $FIVE
      HEL PC
       SAVEVALUE
                 3-9,0,XH
                                      SET COUNTS BACK TO ZERO
      TERMINATE
                                      INITIAL MATRIX CLEARER
      CENERATE
                 3600,,3600,1,4,0
      SAVEVALUE
                 3-0,0,XH
                                      SET COUNTS TO ZERO
       TERMINATE
                               TIMER
******
                                       *******
      GENERATE
                 3600
      TERMINATE
       START
                 1.NP
      RESET
       START
                               OUTPUT
      REPORT
      FJFCT
***** CPSS SIMURATION MODEL FOR ATC VERRAL COMMUNICATIONS SYSTEM ****
                        TRANSPORTATION PROGRAM
                      DEPT. OF CIVIL FNGINEERING
```

```
PRINCETON UNIVERSITY
                             MARCH, 1974
2
      TEXT
                 INPUT PARAMETERS - SECTOR #XH10,2/XXX#
      TFXT
                 (1) AIRCRAFT INTERARRIVAL TIMES: EXPONENTIAL WITH MEA#
W = #XL1,2/XXXX.XXX SECONDS
                 (2) TRANSACTIONS PER AIRCRAFT: SHIFTED NEGATIVE BINOM*
IAL WITH K = #XL3,2/XXX.XXX# AND P = #XL4,2/XXX.XXX
      TEXT
                 (3) TRANSMISSIONS PER TRANSACTION: EMPIRICAL DISTRIBUS
TION
      TFXT
                 (4) TRANSMISSIONS LENGTHS: GAMMA WITH P = #XL2.2/XXX.*
XXXX# AND ALPHA = #XL9,2/XXX.XXXX#
                        (NOTE: GAMMA PARAMETERS DETERMINED FROM EXPECT#
      TFXT
ED ARRIVAL RATE)
      TFXT
                 (5) INTERCOMMUNICATION GAP LENGTHS ARE A FUNCTION OF #
TRANSACTIONS PER AIRCRAFT
# SIMULATION RESPONSE - 2 HOUR ANALYSIS
*
    (1) SECTOR AIRCRAFT LOADING
                 NUMBER OF AIRCRAFT IDENTIFIED IN SECTOR = #51.5/XXXX#
10
10
      TEXT
                 AVERAGE NUMBER OF AIRCRAFT PER SECOND = #$1,3/XXX.XXX*
      TEXT
                 MAXIMUM NUMBER OF AIRCRAFT PER SECOND = 451,8/XXX#
10
    (2) COMMUNICATIONS CHANNEL LOADING
                 AVERAGE CHANNEL UTILIZATION = #F1.2/X.XXX#
10
      TFXT
                 TOTAL NUMBER OF TRANSACTIONS = #F1,3/XXXX#
      TFXT
10
10
      TFXT
                 AVERAGE LENGTH OF TRANSACTIONS = #F1.4/XXX.XXX# SECON*
15
    (3) CHANNEL OUFUFING FFFECTS
10
                 AVERAGE TIME IN QUEUF = #Q1,7/XXXX.XXX# SECONDS
      TFXT
10
      TEXT
                 AVERAGE TIME EXCLUDING ZERO ENTRIES = #01.8/XXXX.XXX##
SECONDS
      TEXT
10
                 TOTAL ENTRIES INTO QUEUF = #01,4/XXXX#
10
      TEXT
                 NUMBER OF ZERO ENTRIES (NON-VAITING) = #01,5/XXXX#
                 PERCENT OF ZERO ENTRIES = #01,6/XXX.X#
      TFXT
10
      TEXT
                 AVERAGE NUMBER OF AIRCRAFT IN QUEUE = #01.3/XXX.XXX#
. 10
      TEXT
10
                 MAXIMUM NUMBER OF AIRCRAFT IN QUEUE = #01,2/XXX#
      FJFCT
>**
                               FND
                                     ******
```

FND

SURPOUT INF EXPON(FY, IX, I1, I2, I3, I4)

R=RANDU(I)

X=(-FX) \* ALOG(R)

IX=X+0.5

RETURN
END

END

10

SUBBRUTINE SUBNBICK, P. NX, 11, 12, 13) IMPLICIT REAL #8(D) RFAL K DP=P DK=K R=RANDU( 11 ) DR=R I = 0 x=0.0 OC LIM= DP # # DK DY=DCUM IF ( DCUM .LT. DR ) GO TO 10 NX=(X+0.5) RETURN 1 = 1 + 1 X=1.0 #1

Dx = x

DY = DY \* ((DK + DX - 1 .0 DO)/DX)

DC (IM = DC (IM + DY \* (1 .0 DO - DP) \* \* DX

IF (CC (IM .LT. DR) GO TO 10

NX = (X + 0 .5)

RET(IPN

FND

. .

SUBROUTINE GAPM(A1, A2, N, Y, II, I2) CH=A.O CL=3.1

XM=A]+A2\*N IF(A2 .LF. 0.) GO TO 1 SD=(CH-XM)/2.5758 GO TO 2 1 SD=(XM-CL)/2.5758

2 X=RNORM(]]) XX=(X\*SO)+XM IF(XY .LF. O.) XX=O. Y=FXP(XX) RFT(IRN END

## BEST AVAILABLE COPY

FUNCTION RNORM(IX)
SIM=0.0
OO 5 [=1,12
SIM=SUM+RANDU(IX)
RNORM=SUM-6.0
RFTURN
END

SHRROUTINE MSSG(P1,P2,N,NTIME,I1,I2) CALL SURGAM(P1,P2,X,I1,I2) NTJME=X+0.5 IF (NTIME .LT. 1) NTIME=1

RFTURN END STARDUTINE SUBGAMIK . A . X . 13 . 14)

RFAL K

C

CCC

C

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20

THIS SUBROUTINE GENERATES RANDOM VARIATES FROM A GAMMA DSN. WITH GENERAL FORM

ν χ ΕΧΡ(-ΦΧ)

CAMMALK

WHEN K IS AN INTEGER, THE GAMMA DSN. IS FOULVALENT TO THE ERLANG DSN., WHICH ARISES AS A SUM DE K EXPONENTIAL VARIATES WITH EXPECTED VALUE 1/4. THEREFORE, THE ERLANG VARIATE X IS EQUAL TO 1/A TIMES THE LOG OF THE PRODUCT OF K PANDOM VARIATES FROM A UNIFORM (0,1) DSN..

WHEN K IS NOT AN INTEGER, AN APPROXIMATE TECHNIQUE MUST BE USED.

LET K=M+O WHERE M IS THE SMALLEST INTEGER CONTAINED IN K AND Q IS THE

REMAINDER. SINCE THE EXPECTED VALUE, VARIANCE, AND THIRD CENTRAL MOMENT

OF GAMMMA VARIATES ARE LINEAR FUNCTIONS OF K, AN APRROXIMATE TECHNIQUE

FOR GENERATING GAMMA VARIATES WITH PARAMETER K IS TO GENERATE A MIXTURE

OF GAMMA VARIATES, CHOOSING M WITH PROB. (1-Q) AND M+1 WITH PROB. Q.

THE APPROXIMATION IMPROVES WITH INCREASING K.

REF. - 'COMPUTER SIMULATION TECHNIQUES'
NAYLOR, BALINTEY, BURDICK, & CHU
JOHN WILEY & SONS, 1966 PP. 87-90

M1=K
M2=M1+1
0=K-FL0AT(M1)
KK=M1
IF (0 .E0. 0.0) G0 T0 10
T=RANDU(I3)
IF (T .LE. 0) KX=M2
TR=1.0
D0 20 I=1,KK
R=RANDU(I4)
TR=TR#R
X=(-ALOG(TR))/A
RFTURN
FND

```
SURROUTINE TIMES(IVALUE, ISAVEF, ISAVEH, IFAC, ISTO, ESTO, JOUE,
      * FOUF, ILOG, ITAR, FTAR, TUSF, TUSFF, FUSE, IMAX, IMAXR, IMAXH, IMAXRH,
      # FSAVFL , IMAXL , FMAXAL)
       INTEGER#2 ISAVEH, ILOG, IUSE, IMAXBH
       REAL #8 FOUF FUSE
       REAL #4 FSTO, FSAVEL , FMA XAL
     DIMENSION IVALUE(6), ISAVEF(2), ISAVEH(2), IFAC(2), ISTO(2), FSTO(2), #IOUE(2), FOUE(2), ILOG(2), ITAB(2), FTAB(2), IUSE(2), IUSE(2), FUSE(2),
      #[MAX(2), [MAXR(2), [MAXH(2), ]MAXRH(2), FSAVFL(2), [MAXL(2), FMAXRL(2)
       INTEGER Z(600) . KEY(3)/1.10.12/
       NIINIT=ISAVEH(1)
       IC = COLUMN NUMBER
       IR=ROW MIJMRFR
       10=1
       ICN=1
       K = 6
       L=1
       nn 200 1JK=1.3
       N=KFY( IJK )
       JK=K=(N-1)+L
       DO 100 IR=1,600
       J=( IMAXH(JK)+2*( ICN*( IR-1)+( IC-1) ))/2
       Z(IR)=IMAXRH(J)
100
       WRITF(NUNIT, 101) Z
101
       FORMAT (4012)
200
       CONTINUE
       RFTURN
       END
```

```
SURPRUTINE PASS2(IVALUE, ISAVEF, ISAVEH, IFAC, ISTR, FSTO, IQUE,
     # FOUF, ILOG. ITAR, FTAR, TUSE, TUSE, FUSE, IMAX, IMAXR, IMAXH, IMAXRH,
     * FSAVEL , IMAXL , FMAXRL ) .
      INTEGER#2 ISAVEH, ILOG, IUSE, IMAXBH
      REAL *8 FOUF FUSE
      REAL #4 FSTO, FSAVEL, FMAXRL
      DIMENSION IVALUE(A), ISAVEE(2), ISAVEH(2), IFAC(2), ISTO(2), ESTO(2),
     * IOUF(2), FOUF(2), ILOG(2), ITAB(2), FTAB(2), IUSF(2), IUSF(2), FUSE(2),
     #1MAX(2),1MAXR(2),1MAXH(2),1MAXFH(2),FSAVFL(2),1MAXL(2),FMAXRL(2)
      INTEGER Z(1200),N(9),KEYN(9)/3,3,4,5,4,6,7,8,9/
      INTEGER KEYM(9)/2,3,4,5,6,7,8,9,11/
      NINIT=ISAVEH(2)
      IC=1
      IC N=1
      K=6
      L=1
      nn 100 I=1.9
      KN=KFYN(])
100
      N(I) = ISAVEH(KN)
      WPITF(NUNIT, 101) N
101
      FORMAT(915)
      nn 200 1=1,9
      KM=KFYM(I)
      NR = N( I)
      JK=K = (KM-1)+L
      DO 300 1R=1,NR
      J=( 1MA XH( JK )+2*(ICN*(IR-1-1+(IC-1)))/2
300
      Z(JR) = IMAXAH(J)
      WRITF(NUNIT, 102) (Z(M), M=1, NR)
102
      FORMAT(1615)
      CONTINUE
200
      RETURN
      END
```

BEJ WAILBIL OP

SUBROUTINE EXPON(FY, IX, I1, 12, 13, 14)
R=RAND(I(I1)
X=(-FX) \*ALDG(R)
IX=X+0.5
RETURN
END

FUNCTION RANDU( 1X )

1Y=1X + 65539

IF( 1Y) 5,6,6

1Y=1Y+2147483647+1

RANDU=1Y

RANDU=RANDU+.4656613F-9

IX=1Y

RETURN

END

10

SUBROUTINE SUBNBICK, P. NX, 11, 12, 131 IMPLICIT RFAL #8(D) RFAL K DP=P DK=K R=RANDU(11) DR=R 1=0 X=0.0 DC LIM= DP # + DK DY=DCUM IF ( DCIM .LT. DR ) GO TO 10 NX=(X+0.5) RETURN [=]+] X=1.0+1

DX=X
DY=DY\*((DK+DX-1.000)/DX)
DC IM=DC IM+DY\*(1.000-DP)\*\*DX
IF (DC IM .LT. DR) GO TO 10
NX=(X+0.5)
RFT(IRN
FND

SUBROUTINE CAPMIA1, A2, N, Y, 11, 12) C11=6.0 CL=3.1 XM=4]+42\*N IF(42 .LF. 0.) GO TO 1 SD=(CH-XM)/2.5758 GD TD 2 1 SP=( XM-CL 1/2.5758 2 X=RMCRM(11) XX=(X#SD)+XM IF(XY .LF. O.) XX=O. Y=FXP(XX) RETURN END

FUNCTION RNCRM(IX) SIIM=0.0 nn 5 I=1,12 SIIM=SIIM+RANDUI( 1X ) . RMORM=SIM-6.0 RETURN END

SI WAILBIL ORY SUBROUTINE MSSG(P1,P2,N,NTIME,11,12) CALL SURGAM(P1,P2,X,11,12) NT JMF=X+0.5 IF (NTIME .LT. 1) NTIME=1

RFTIIRN END

SIRROUTINE SURGAMIK, A. X, 13, 14)

RFAL K

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THIS SUBROUTINE GENERATES RANDOM VARIATES FROM A GAMMA DSN. WITH GENERAL FORM

A X EXP(-AX)

#### GAMMA(K)

WHEN K IS AN INTEGER, THE GAMMA OSN. IS FOLITVALENT TO THE ERLANG OSN., WHICH ARISES AS A SUM OF K EXPONENTIAL VARIATES WITH EXPECTED VALUE I/A. THEREFORE, THE ERLANG VARIATE X IS EQUAL TO 1/A TIMES THE LOG OF THE PRODUCT OF K PANDOM VARIATES FROM A UNIFORM (0.1) DSN..

WHEN K IS NOT AN INTEGER, AN APPROXIMATE TECHNIQUE MUST BE USED. LET K=M+O WHERE M IS THE SMALLEST INTEGER CONTAINED IN K AND Q IS THE REMAINDER. SINCE THE EXPECTED VALUE, VARIANCE, AND THIRD CENTRAL MOMENT OF GAMMMA VARIATES ARE LINEAR FUNCTIONS OF K, AN APPROXIMATE TECHNIQUE FOR GENERATING GAMMA VARIATES WITH PARAMETER K IS TO GENERATE A MIXTURE OF GAMMA VARIATES, CHOOSING M WITH PROB. (1-Q) AND M+1 WITH PROB. Q. THE APPROXIMATION IMPROVES WITH INCREASING K.

REF. - 'COMPUTER SIMULATION TECHNIQUES'
NAYLOR, BALINTEY, BURDICK, & CHU
JOHN WILEY & SONS, 1966 PP. 87-90

M1=K
M2=M1+1
O=K-FLOAT(M1)
KK=M1
IF (O .FO. O.O) GO TO 10
T=RANDU(13)
IF (T .LE. O) KK=M2
TR=1.0
OO 20 I=1,KK
R=RANDU(14)
TR=TR\*R
X=(-ALOG(TR))/A
RFTURN

```
SURROUTINE TIMES(IVALUE, ISAVEF, ISAVEH, IFAC, ISTO, FSTO, IQUE,
     * FOUF, ILOG, ITAR, FTAR, TUSE, TUSEF, FUSE, IMAX, IMAXR, IMAXH, IMAXRH,
     * FSAVEL , IMAXL , FMAXRL)
      INTEGER#2 ISAVEH, ILOG, IUSE, IMAX BH
      REAL #R FOUF , FUSE
      RFAL #4 FSTO, FSAVFL, FMAXAL
      DIMENSION IVALUE(6). ISAVEF(2). ISAVEH(2). IFAC(2). ISTO(2). FSTO(2).
     #IOUF(2),FOUF(2),ILOG(2),ITAB(2),FTAB(2),IUSE(2),IUSE(2),FUSE(2),
     #IMAX(2), IMAXA(2), IMAXH(2), IMAXAH(2), FSAVEL(2), IMAXL(2), FMAXAL(2)
      INTEGER 2(600), KEY(3)/1,10,12/
      NIIN IT= ISAVFH(1)
      IC = COLUMN NUMBER
      IR=ROW MIMBER
      IC=1
      1CN=1
      K=6
      L = 1
      DO 200 1JK=1.3
      N=KFY(IJK)
      JK=K # (N-1)+L
      DO 100 IR=1,600
      J=(IMAXH(JK)+2*(ICN*(IR-1)+(IC-1)))/2
100
      Z(IR) = IMAXRH(J)
      WRITF(NUMIT, 101) Z
101
      FORMAT(4012)
      CONTINUE
200
                    ST WHART ON
      RETURN
      END
```